MTH 309

Additional Problems on Modular Arithmetic (Sec 4.1)

1. Verify each of the following.

   (a) $(5 \oplus_8 6) \oplus_8 7 = 5 \oplus_8 (6 \oplus_8 7)$

   (b) $(5 \odot_8 6) \odot_8 7 = 5 \odot_8 (6 \odot_8 7)$

   (c) $(5 \odot_8 6) \oplus_8 (5 \odot_8 7) = 5 \odot_8 (6 \oplus_8 7)$

   (d) 27 and 32 are additive inverses in $\mathbb{Z}_{59}$

   (e) 27 and 32 are multiplicative inverses in Z_{863}

2. Use the additive inverse property to find the additive inverse in $\mathbb{Z}_{35}$ of each of the following:

   (a) 17

   (b) 1

   (c) 34

   (d) 0

3. For each of the following values of m, use the multiplication table for $\mathbb{Z}_m$ to determine the elements of $\mathbb{Z}_m$ that have multiplicative inverses and give the multiplicative inverse.

   (a) $m = 5$

   (b) $m = 6$

4. Prove that for all $m \in \mathbb{Z}_{>0}$, the multiplicative inverse of $m - 1$ in $\mathbb{Z}_m$ is $m - 1$.

5. Prove each of the following identities for $a, b, c \in \mathbb{Z}_m$:

   (a) $(a \odot_m b) \odot_m c = a \odot_m (b \odot_m c)$ (associativity)

   (b) $(a \odot_m b) \oplus_m (a \odot_m c) = a \odot_m (b \oplus_m c)$ (distributivity)