MTH 309

Additional Problems for Sec 2.3

1. Let $1 \leq k \leq n$. Use a bijection to show that the number of subsets of $\{1, 2, \ldots, n\}$ of size k that contain n is equal to the number of subsets of $\{1, 2, \ldots, n-1\}$ of size $k-1$. (Describe the domain and codomain with set builder notation and give the rule.)

2. Let $S = \{1, 2, \ldots, n\}$.

   (a) Find a bijection from the set of subsets of S that contain an even number of elements to the set of subsets of S that contain an odd number of elements. (Describe the domain and codomain with set builder notation and give the rule.)

   (b) Can you conclude from (a) that the number of subsets of S of even cardinality equals the number of subsets of S of odd cardinality?

   (c) Find a formula for the number of subsets of S that have an even number of elements.