

MTH 309

Additional Problems for Sec. 2.3

1. Let $g : A \rightarrow B$ and $f : B \rightarrow C$ be functions. Justify your answer on each of the following.
 - (a) If $f \circ g$ is one to one, does it follow that g is one to one?
 - (b) If $f \circ g$ is one to one, does it follow that f is one to one?
 - (c) If $f \circ g$ is onto, does it follow that g is onto?
 - (d) If $f \circ g$ is onto, does it follow that f is onto?
2. For each of the following bijections, find its inverse. (Be sure to include domain, codomain and rule.)
 - (a) $f : \mathbb{R} \rightarrow \mathbb{R}$, where $f(x) = 7x - 4$.
 - (b) Let $\mathcal{A} = \{0, 1\}$. $f : \mathcal{A}_n \rightarrow \mathcal{A}_n$, where $f(a_1 a_2 \dots a_n) = a_n a_1 a_2 \dots a_{n-1}$.
 - (c) $f : \{T \in \mathcal{P}(\{1, 2, \dots, n\}) \mid n \in T\} \rightarrow \mathcal{P}(\{1, 2, \dots, n-1\})$, where $f(T) = T - \{n\}$.
3. Let $0 \leq k \leq n$. Find a bijection from the set of subsets of size k of the set $U = \{1, 2, \dots, n\}$ to the set of subsets of size $k+1$ of the set $V = \{1, 2, \dots, n+1\}$ that contain the integer $n+1$. (Describe the domain and codomain by using set builder notation and express the rule by using unions.)
4. Let $S = \{1, 2, \dots, n\}$.
 - (a) Find a bijection from the set of subsets of S of even cardinality to the set of subsets of S of odd cardinality. (Describe the domain and codomain with set builder notation and give the rule.)
 - (b) Can you conclude from (a) that the number of subsets of S of even cardinality equals the number of subsets of S of odd cardinality?
 - (c) Find a formula for the number of subsets of S that have an even number of elements.