

MTH 309

Additional Problems for Sec 2.3

1. Let $\mathcal{A} = \{0, 1\}$ and let $\mathcal{A}^*$ be the set of bit strings.

$$f : \mathcal{A}^* \rightarrow \mathbb{N}$$

$$f(w) = \text{the number of 1's in } w$$

Find $f(110100)$.

2. $f : \mathcal{P}(\{0, 1, \dots, n\}) \rightarrow \mathcal{P}(\{1, 2, \dots, n+1\})$

$$f(S) = \{x \in \{1, 2, \dots, n+1\} \mid x-1 \in S\}$$

Find

(a) $f(\{2, 5, 9\})$

(b) $f(\emptyset)$

(c) $f(\{0, 1, \dots, n\})$

3. $f : \mathcal{P}(\{1, 2, \dots, 20\}) \rightarrow \mathcal{P}(\{1, 2, \dots, 21\})$

$$f(S) = S \cup \{21\}.$$

Find

(a) $f(\{2, 5, 9\})$

(b) $f(\emptyset)$

(c) $f(\{1, \dots, 20\})$

4. Let $f : \mathcal{P}(\{1, 2, \dots, n\}) \rightarrow \{0, 1\}_n$

$$f(T) = w_1 w_2 \cdots w_n \text{ where } w_i = \chi(i \in T) \text{ for all } i \in \{1, 2, \dots, n\}.$$

For $n = 7$, find

(a) $f(\{2, 4, 5\})$

(b) $f(\emptyset)$

(c) $f(\{1, 2, 3, 4, 5, 6, 7\})$

5. Decide which of the following are functions. For those that are functions, determine whether the function is onto and determine whether the function is one to one.

(a) $f : \{1, 2, 3\} \rightarrow \{2, 3, 4\}$

$$f(1) = 3, \quad f(2) = 4, \quad f(3) = 4$$

(b) $f : \mathbb{N} \rightarrow \mathbb{N}$

$$f(n) = n + 3$$

(c) $f : \mathbb{Z} \rightarrow \mathbb{Z}$

$$f(n) = n + 3$$

(d) $f : \mathbb{N} \rightarrow \mathbb{N}$

$$f(n) = n - 3$$

(e) Let $\mathcal{A} = \{0, 1\}$ and let $\mathcal{A}^*$ be the set of bit strings.

$$f : \mathcal{A}^* \rightarrow \mathbb{N}$$

$$f(w) = \text{the number of 1's in } w$$

(f) $f : \mathbb{Q} \rightarrow \mathbb{Q}$

$$f(p/q) = p$$

(g) $f : \mathcal{P}(\{0, 1, \dots, n\}) \rightarrow \mathcal{P}(\{1, 2, \dots, n+1\})$

$$f(S) = \{x \in \{1, 2, \dots, n+1\} \mid x-1 \in S\}$$

(h) $f : \mathcal{P}(\{1, 2, \dots, n\}) \rightarrow \mathcal{P}(\{1, 2, \dots, n+1\})$

$$f(S) = S \cup \{n+1\}$$

(i) $f : \mathbb{Z}_{>0} \rightarrow \mathbb{Q}$

$$f(n) = 1/n$$