ADDITIONAL PROBLEMS FOR MTH 532

Problem A1. Let $\mathrm{GL}(n, \mathbb{C})$ consist of complex $n \times n$ matrices A such that $\det A \neq 0$. Show that $\mathrm{GL}(n, \mathbb{C})$ is a Lie group of dimension $2n^2$. It is called the (complex) general linear group.

Problem A2. Let $\mathrm{SL}(n, \mathbb{C})$ consist of complex $n \times n$ matrices A such that $\det A = 1$. Show that $\mathrm{SL}(n, \mathbb{C})$ is a Lie group of dimension $2n^2 - 2$. It is called the (complex) special linear group.

Problem A3. Let $U(n)$ consist of complex $n \times n$ matrices A such that $A\bar{A}^t = E$. Here, $\bar{A}$ is obtained from A by replacing each entry of A with its complex conjugate. Show that $U(n)$ is a Lie group of dimension n^2. It is called the unitary group. [Hint: observe that $C = A\bar{A}^t$ is a Hermitian matrix, meaning that $\bar{C} = C^t$, and that all Hermitian matrices form a vector space of real dimension n^2].

Problem A4. Show that $U(1)$ is diffeomorphic, as a manifold, to the circle. Let $SU(2)$ be the subgroup of $U(2)$ consisting of matrices with determinant 1. Show that $SU(2)$ is diffeomorphic, as a manifold, to the 3-sphere.