Analysis Preliminary Exam

The exam consists of 10 questions. A few may ask you to state and/or prove a theoretical result. The problems are in the same range of difficulty as the ones in the MTH533-534 tests but they do not follow one particular textbook.

- Axioms and properties of natural, integer, rational and irrational numbers.
- Order. Completeness. Definition and properties of sup and inf.
- Convergence: definition with $\epsilon - \delta$ and sequences. Monotone sequences, bounded sequences, $\limsup$ and $\liminf$.
- Continuity: definition, properties, the extreme value theorem, intermediate value theorem.
- Uniform continuity: definition with $\epsilon - \delta$ and with sequences. Lipschitz continuity and examples.
- Compact sets: sequential compactness equivalent to closed and bounded (Heine-Borel Theorem).
- Differentiability: first order, higher order differentiability, mean value theorem, Taylor formula, first and second derivative tests.
- Riemann integral: Upper Darboux sum, Lower Darboux sum. Definition of the integral. Arhimedean theorem.
- Definition of the logarithm, exponential, properties.
- Series. Convergence criteria. Cauchy criterion. Absolute convergence. Weierstrass Criterion.
- Sequences and series of functions. Uniform convergence. Power series. The binomial formula.
- The Stone -Weierstrass theorem on uniform approximation of continuous functions on compacts by polynomials. Statement only.
- The $\mathbb{R}^d$ space. Norms. Open, closed sets. Compact sets in $\mathbb{R}^d$.
- Continuous functions. Absolute continuity. The extreme value theorem.
- Connected sets and pathwise connectedness. Properties of continuous functions.
- Differentiability. Partial derivatives. The differential. The Gradient. Directional derivatives. Mean value theorem.
- Taylor series in $\mathbb{R}^d$. The first and second approximations of a differentiable function. Critical points. The second derivative test.
- Applications of the Chain rule. Smooth surfaces in $\mathbb{R}^n$. Tangent plane to a surface.
- The Inverse function theorem. Statement and applications.
- The Implicit function theorem. Statement and applications.
- Lagrange multipliers method.