

Here are some basic facts that we needed in class:

**Facts that are wrong.**

- $\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$
- $\frac{a}{a+b} \neq \frac{1}{1+b}$
- $\sin(ab) \neq \sin(a)\sin(b)$
- $\sin(2x) \neq 2\sin(x)$

**Facts about exponents.**

- $a^b \cdot a^c = a^{b+c}$
- $(a^b)^c = a^{bc}$
- $a^{-b} = \frac{1}{a^b}$
- $\frac{a^b}{a^c} = a^{b-c}$

**Facts about limits.**

- $1/n \rightarrow 0$  as  $n \rightarrow \infty$
- If  $f(x) \rightarrow +\infty$  or  $f(x) \rightarrow -\infty$  as  $x \rightarrow a$  then  $1/f(x) \rightarrow 0$  as  $x \rightarrow a$
- If  $-1 < a < 1$  then  $a^n \rightarrow 0$  as  $n \rightarrow \infty$
- e.g.  $(6/7)^n \rightarrow 0$  as  $n \rightarrow \infty$  because  $-1 < 6/7 < 1$
- If  $a > 1$  then  $a^n \rightarrow \infty$  as  $n \rightarrow \infty$ . If  $a < -1$  then  $\lim_{n \rightarrow \infty} a^n$  does not exist.
- If  $a > 0$  then  $t^a \rightarrow \infty$  as  $t \rightarrow \infty$ , hence  $1/t^a \rightarrow 0$  as  $t \rightarrow \infty$
- e.g.  $\sqrt{t} = t^{1/2} \rightarrow \infty$  as  $t \rightarrow \infty$  because  $1/2 > 0$
- If  $a > 0$  then  $a/x \rightarrow +\infty$  as  $x \rightarrow 0^+$  and  $a/x \rightarrow -\infty$  as  $x \rightarrow 0^-$
- If  $a < 0$  then  $a/x \rightarrow -\infty$  as  $x \rightarrow 0^+$  and  $a/x \rightarrow +\infty$  as  $x \rightarrow 0^-$

**Facts about trigonometry.**

- $-1 \leq \sin x \leq 1$  for all  $x$
- $0 \leq \sin^2 x \leq 1$  for all  $x$