Math 161 **Summer 2024**
Exam 2 **Drew Armstrong**

---

1. Compute the derivative of $f(x) = \sqrt[3]{x}$.

Since $f(x) = \sqrt[3]{x} = x^{1/3}$ we use the power rule to get

$$f'(x) = \frac{1}{3} \cdot x^{1/3-1} = \frac{1}{3} \cdot x^{-2/3}.$$

2. Compute the derivative of $f(x) = (x^3 + 2)^5$.

We use the power rule and the chain rule to get

$$f'(x) = 5(x^3 + 2)^4 \cdot (x^3 + 2)' = 5(x^3 + 2)^4(3x^2 + 0).$$

3. Compute $\dfrac{dy}{d\theta}$ where $y = \dfrac{\cos\theta}{\sin\theta}$.

We use the quotient rule together the rules $\frac{d}{d\theta}(\sin\theta) = \cos\theta$ and $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$ to get

$$\begin{aligned}
\frac{dy}{d\theta} &= \frac{d}{d\theta}\left(\frac{\cos\theta}{\sin\theta}\right) \\
&= \frac{\sin\theta \cdot \frac{d}{d\theta}(\cos\theta) - \cos\theta \cdot \frac{d}{d\theta}(\sin\theta)}{(\sin\theta)^2} \\
&= \frac{\sin\theta \cdot (-\sin\theta) - \cos\theta \cdot (\cos\theta)}{(\sin\theta)^2} \\
&= \frac{-(\sin\theta)^2 - (\cos\theta)^2}{(\sin\theta)^2} \\
&= -\frac{1}{(\sin\theta)^2}.
\end{aligned}$$

Remark: You can call this $-\csc^2\theta$ if you want. I prefer not to.

4. Compute the derivative of $g(u) = u \cdot \sqrt{u^2 + 1}$.

We use the product rule, the chain rule and the power rule to get

$$\begin{aligned}
g'(u) &= (u)' \cdot \sqrt{u^2 + 1} + u \cdot (\sqrt{u^2 + 1})' \\
&= (1) \cdot \sqrt{u^2 + 1} + u \cdot \frac{1}{2}(u^2 + 1)^{-1/2} \cdot (2u)
\end{aligned}$$

5. Find the **slope of the tangent line** to the curve $y^3 + xy = 3$ at the point $(x, y) = (2, 1)$.

We use implicit differentiation to compute dy/dx:

$$\begin{aligned}
y^3 + xy &= 3 \\
\frac{d}{dx}\left(y^3 + xy\right) &= \frac{d}{dx}3
\end{aligned}$$

$$
\begin{aligned}
\frac{d}{dx}\left(y^3\right) + \frac{d}{dx}(xy) &= 0 \\
3y^2 \cdot \frac{dy}{dx} + \left(x \cdot \frac{dy}{dx} + 1 \cdot y\right) &= 0 \\
\left(3y^2 + x\right) \cdot \frac{dy}{dx} &= -y \\
\frac{dy}{dx} &= \frac{-y}{3y^2 + x}.
\end{aligned}
$$

This is the slope of the tangent line at a general point (x, y) on the curve. The slope of the tangent at the point $(x, y) = (2, 1)$ is

$$
\frac{dy}{dx} = \frac{-(1)}{3(1)^2 + 2} = -\frac{1}{5}.
$$

Remark: We can go further to compute the slope of the tangent line at the point $(2, 1)$. The point-slope formula gives

$$
\begin{aligned}
\frac{y-1}{x-2} &= -\frac{1}{5} \\
y - 1 &= -\frac{1}{5}(x-2) \\
y &= 1 - \frac{1}{5}x + \frac{2}{5} \\
y &= -\frac{1}{5}x + \frac{7}{5}
\end{aligned}
$$

Here is a picture produced by desmos (note that the red curve has two pieces):