**Math 161** **Summer 2024**
**Exam 1** **Drew Armstrong**

Compute the following limits. In some cases the answer may be $+\infty$ or $-\infty$.

**1.** $\displaystyle\lim_{x\to 1} \frac{x-1}{x^2-3x+2}$

This limit has the indeterminate form "0/0" so we need a trick. We factor the denominator and then cancel:

$$\lim_{x\to 1} \frac{x-1}{x^2-3x+2} = \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(x-2)} = \lim_{x\to 1} \frac{1}{x-2} = \frac{1}{-1} = -1.$$

**2.** $\displaystyle\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}$

This limit has the indeterminate form "0/0" so we need a trick. We multiply the numerator and denominator by the conjugate expression to "rationalize the numerator":

$$\begin{aligned}
\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} & \lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} \cdot \frac{\sqrt{9+h}+3}{\sqrt{9+h}+3} \\
&= \lim_{h\to 0} \frac{(9+h)-9}{h\left(\sqrt{9+h}+3\right)} \\
&= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{9+h}+3\right)} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3} \\
&= \frac{1}{\sqrt{9}+3} \\
&= \frac{1}{3+3} \\
&= \frac{1}{6}.
\end{aligned}$$

**3.** $\displaystyle\lim_{\theta\to 0} \frac{\sin\theta}{\theta\cos\theta}$

Here we use the fact that $(\sin\theta)/\theta \to 1$ as $\theta \to 0$:

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta\cos\theta} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \cdot \frac{1}{\cos\theta} = 1 \cdot \frac{1}{\cos 0} = 1 \cdot 1 = 1.$$

**4.** $\lim_{t\to 1} \dfrac{t-2}{(t-1)^2}$

We note that $(t-1)^2$ is a tiny positive number when $t$ approaches 1 **from either direction**. Hence

$$\lim_{t\to 1} \frac{t-2}{(t-1)^2} = \frac{1-2}{\text{tiny positive number}} = \frac{-1}{\text{tiny positive number}} = -\infty.$$

**5.** $\lim_{n\to\infty} \dfrac{2n^3+1}{n^3+n}$

When $n \to \infty$ we should divide numerator and denominator by the highest power of $n$ that occurs, in this case $n^3$. Then since $1/n \to 0$ and $1/n^2 \to 0$ we have

$$\begin{aligned}
\lim_{n\to\infty} \frac{2n^3+1}{n^3+n} &= \lim_{n\to\infty} \frac{(2n^3+1)/n^3}{(n^3+n)/n^3} \\
&= \lim_{n\to\infty} \frac{2+\frac{1}{n^3}}{1+\frac{1}{n^2}} \\
&= \frac{2+0}{1+0} \\
&= 2.
\end{aligned}$$